The Description of F_2 at Very High Parton Densities

M. B. Gay Ducati [1,*] *and* *V. P. Gonçalves* [2,**]

[1] *Instituto de Física, Univ. Federal do Rio Grande do Sul*
Caixa Postal 15051, 91501-970 Porto Alegre, RS, BRAZIL
[2] *Instituto de Física e Matemática, Univ. Federal de Pelotas*
Caixa Postal 354, 96010-090 Pelotas, RS, BRAZIL

Abstract: The behavior of the structure function at high energies (high densities) is directly associated to the gluon distribution. In this paper we analyze the asymptotic behavior of the structure function at very high densities considering a leading and a higher twist relation between these two quantities. We verify that (a) if the leading twist relation between the structure function and the gluon distribution is used, we recover the black disc limit, and (b) a softer behavior is predicted if a higher twist relation between F_2 and xG is considered. While the first behavior is well-established by the current high density approaches, the higher twist behavior is a new result. In both cases, the F_2 structure function unitarizes and the Froissart boundary is not violated in the asymptotic regime of high density QCD.

[0,*]E-mail:gay@if.ufrgs.br
[0,**]E-mail:barros@ufpel.tche.br

1 Introduction

The physics of high-density QCD has become an increasingly active subject of research, both from experimental and theoretical points of view. In deep inelastic scattering (DIS) the parton high density regime corresponds to the small x region and represents the challenge of studying the interface between the perturbative and nonperturbative QCD, with the peculiar feature that this transition is taken in a kinematical region where the strong coupling constant α_s is small. By the domain of perturbative QCD we mean the region where the parton picture has been developed and the separation between the short and long distance contributions (the collinear factorization) is made possible by the use of the operator product expansion (OPE). The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [1] are the evolution equations in this kinematical region. These equations are valid at leading twist, i.e. at a large value of the photon virtuality Q^2, where a subclass of all possible Feynmann graphs are dominant and the hard coefficient function is connected to the proton by only two parton lines (For more details see e.g. Ref. [2]). For small values of Q^2, this picture has corrections predicted by the OPE that contribute at relative order $\mathcal{O}(1/Q^2)$ and beyond $[\mathcal{O}(\frac{1}{Q^2})^n, \ n = 2, 3, ...]$. These are commonly called higher twist corrections.

In the limit of small values of x ($< 10^{-2}$), on the other hand, one expects to see new features inside the nucleon: the density of gluons and quarks become very high and an associated new dynamical effect is expected to stop the further growth of the structure functions. In particular, for a fixed hard scale $Q^2 \gg \Lambda_{QCD}^2$, the OPE eventually breaks down at sufficiently small x [3]. Ultimately, the physics in the region of high parton densities will be described by nonperturbative methods, which is still waiting for a satisfactory solution in QCD. However, the transition from the moderate x region towards the small x limit may possibly be accessible in perturbation theory, and, hence, allows us to test the ideas about the onset of nonperturbative dynamics.

At this moment, there are several approaches in the literature that propose distinct evolution equations for the description of the gluon distribution in high density limit [5, 6] [7, 8]. In general these evolution equations resum a class of higher twist diagrams dominant in this kinematical region that contain powers of the function $\kappa(x, Q^2) \equiv \frac{3\pi^2 \alpha_s A}{2Q^2} \frac{xg(x,Q^2)}{\pi R_A^2}$, which represents the probability of gluon-gluon interaction inside the parton cascade. Moreover,

these equations match (a) the DLA limit of the DGLAP evolution equation in the limit of low parton densities ($\kappa \rightarrow 0$); (b)the GLR equation and the Glauber-Mueller formula as first terms of the high density effects. The main differences between these approaches occurs in the limit of very large densities, where all powers of κ should be resummed. Although the complete demonstration of the equivalence between these formulations in the region of large κ is still an open question, some steps in this direction were given recently [9, 10].

Our goal in this paper is not to demonstrate the equivalence between the distinct approaches, but to analyze the relation between the gluon distribution xG and the F_2 structure function in the asymptotic regime of very high densities (energies). In general, a leading twist relation is used to relate these quantities and the solutions of the evolution equations used as input in the calculations. However, as discussed above, the high density approaches resum a class of higher twist diagrams, which implies that we should careful in our predictions for the quantities which would be measured. Here we study the behavior of F_2, predicted by a leading and a higher twist relation, at large densities, where analytical calculations are possible. In this regime we expect the blackness of the cross section, as predicted by Gribov many years ago [11]. We verify that this behavior is recovered by the current high density approaches, if a leading twist relation between F_2 and xG is used. However, a softer behavior is obtained when we consider that this relation is modified by the higher twist terms associated to the high density, demonstrating the importance of this contribution. In both cases, the Froissart boundary is not violated.

The paper is organized as follows: in the next section we briefly review the derivation of the leading and higher twist relations between F_2 and xG, as well as the black disc limit. After, in section 3, we derive the asymptotic solution of the AGL equation at fixed and running α_s and compare this solution with the predictions of the McLerran - Venugopalan high density approach. Finally, in section 4, we present our results and conclusions.

2

2   The F_2 Structure Function and the Black Disc Limit

We start from the space-time picture of the ep processes [11]. The deep inelastic scattering $ep \to e+X$ is characterized by a large electron energy loss ν (in the target rest frame) and an invariant momentum transfer $q^2 \equiv -Q^2$ between the incoming and outgoing electron such that $x = Q^2/2m_N\nu$ is fixed. In terms of Fock states we then view the ep scattering as follows: the electron emits a photon ($|e > \to |e\gamma >$) with $E_\gamma = \nu$ and $p_{t\gamma}^2 \approx Q^2$, after the photon splits into a $q\bar{q}$ ($|e\gamma > \to |eq\bar{q} >$) and typically travels a distance $l_c \approx 1/m_N x$, referred as the coherence length, before interacting in the nucleon. For small x (large s, where $\sqrt{s}$ is $\gamma^* p$ center-of-mass energy), the photon converts to a quark pair at a large distance before it interacts to the target. Consequently, the space-time picture of the DIS in the target rest frame can be viewed as the decay of the virtual photon at high energy (small x) into a quark-antiquark pair long before the interaction with the target. The $q\bar{q}$ pair subsequently interacts with the target. In the small x region, where $x \ll \frac{1}{2mR}$, the $q\bar{q}$ pair crosses the target with fixed transverse distance r_t between the quarks. Following Gribov [11], we may write a double dispersion relation for the forward $\gamma^* p$ elastic amplitude A, related to the total cross section by the optical theorem ($Im\, A = s\sigma(s, Q^2)$), and obtain for fixed s

$$\sigma(s, Q^2) = \sum_q \int \frac{dM^2}{M^2 + Q^2} \frac{dM'^2}{M'^2 + Q^2} \rho(s, M^2, M'^2) \frac{1}{s} Im\, A_{q\bar{q}+p}(s, M^2, M'^2) \ , \quad (1)$$

where M and M' are the invariant masses of the incoming and outgoing $q\bar{q}$ pair. If we assume that the forward $q\bar{q} + p$ scattering does not change the momentum of the quarks then $A_{q\bar{q}+p}$ is proportional to $\delta(M^2 - M'^2)$, and (1) becomes

$$\sigma(s, Q^2) = \sum_q \int \frac{dM^2}{(M^2 + Q^2)^2} \rho(s, M^2) \sigma_{q\bar{q}+p}(s, M^2) \ , \quad (2)$$

where the spectral function $\rho(s, M^2)$ is the density of $q\bar{q}$ states, which may be expressed in terms of the $\gamma^* \to q\bar{q}$ matrix element [12]. Using that $M^2 = (k_t^2 + m_q^2)/[z(1-z)]$, where k_t and z are the transverse and longitudinal momentum components of the quark with mass m_q, we can express the integral over the

3

mass M of the $q\bar{q}$ in terms of a two-dimensional integral over z and k_t. Instead of k_t, it is useful to work with the transverse coordinate r_t (impact parameter representation), which is the variable Fourier conjugate to k_t, resulting [13]

$$F_2(x, Q^2) = \frac{Q^2}{4\pi\alpha_{em}}\sigma(s, Q^2) = \frac{Q^2}{4\pi\alpha_{em}} \int dz \int \frac{d^2r_t}{\pi} |\Psi(z, r_t)|^2 \sigma_{q\bar{q}}(z, r_t) , \quad (3)$$

where

$$|\Psi(z, r_t)|^2 = \frac{6\alpha_{em}}{(2\pi)^2} \sum_i^{n_f} e_i^2 \{[z^2 + (1-z)^2]\epsilon^2 K_1(\epsilon r_t)^2 + m_i^2 K_0(\epsilon r_t)^2\} . \quad (4)$$

The photon wave function $\Psi(z, r_t)$ is simply the Fourier transform of the matrix element for the transition $\gamma^* \rightarrow q\bar{q}$. Moreover, α_{em} is the electromagnetic coupling constant, $\epsilon^2 = z(1-z)Q^2 + m_i^2$, m_i is the quark mass, n_f is the number of active flavors, e_f^2 is the square of the parton charge (in units of e), $K_{0,1}$ are the modified Bessel functions and z is the fraction of the photon's light-cone momentum carried by one of the quarks of the pair. In the leading $\log(1/x)$ approximation we can neglect the change of z during the interaction and describe the cross section $\sigma^{q\bar{q}}(z, 4/r_t^2)$ as a function of the variable x. Considering only light quarks ($i = u, d, s$) F_2 can be expressed by [14]

$$F_2(x, Q^2) = \frac{1}{4\pi^3} \sum_{u,d,s} e_i^2 \int_{\frac{1}{Q^2}}^{\frac{1}{Q_0^2}} \frac{d^2r_t}{\pi r_t^4} \sigma_{q\bar{q}}(x, r_t) . \quad (5)$$

Using that $\sigma_{q\bar{q}} = \frac{C_F}{C_A}(3\alpha_s(\frac{4}{r_t^2})/4)\pi^2 r_t^2 xG(x, \frac{4}{r_t^2})$ [14], where $xG(x, \frac{4}{r_t^2})$ is the nucleon gluon distribution, we get

$$F_2(x, Q^2) = \frac{2\alpha_s}{9\pi} \int_{Q_0^2}^{Q^2} \frac{dQ^2}{Q^2} xG(x, Q^2) . \quad (6)$$

The above expression explicit the direct dependence of the F_2 structure function with the behavior of the gluon distribution, which is related to the QCD dynamics at high energies. In the linear regime, where the high densities can be disregarded, the small x behavior of the gluon distribution is given by the solutions of the DGLAP [1] and/or BFKL [4] evolution equations. The common feature of these equations is the steep increase of xG as x decreases.

This steep increase cannot persist down to arbitrary low values of x since it violates a fundamental principle of quantum theory, *i.e.* the unitarity. In the context of relativistic quantum field theory of the strong interactions, unitarity implies that the cross section cannot increase with increasing energy s above $log^2 s$: the Froissart's theorem [15]. In the next section we will consider the modifications in the behavior of the gluon distribution associated to the high density effects. The presence of a high parton density implies a slow growth of xG at high energies, and consequently from Eq. (6), of the structure function. However, the Eq. (6) is a leading twist relation which will eventually breaks down when we consider the higher twist terms in the evolution.

We now consider that the relation between the structure function and the gluon distribution is modified by the particular type of higher twist terms associated with the high density corrections (See also [16] for a similar calculation in the infinite momentum frame). In general, the higher twist contributions should be significant at small x and Q^2, implying large perturbative corrections to the conventional leading twist relations in this kinematical region. We estimated these corrections considering the s-channel unitarity constraint in the interaction cross section of the quark-antiquark pair with the target [17]. In this case the structure function is given by

$$F_2(x, Q^2) = \frac{1}{2\pi^3} \sum_{f=u,d,s} e_f^2 \int_{\frac{1}{Q^2}}^{\frac{1}{Q_0^2}} \frac{d^2 r_t}{\pi r_t^4} \int d^2 b_t \{1 - e^{-\frac{1}{2}\sigma^{q\bar{q}}(x,4/r_t^2)S(b_t)}\} \ . \qquad (7)$$

A similar expression was used in Ref. [18] for a phenomenological analysis of the ep process, where the geometrical structure of the collision (the $\vec{b}_t$ dependence) was disregarded and $xg \propto x^{-\lambda}$ ($\lambda > 0$) was assumed, resulting a very good description of the HERA data. The main point of the above expression is that this resums a large class of higher twist terms, as demonstrated in Ref. [19].

The use of a Gaussian parameterization for the nucleon profile function $S(b_t) = \frac{1}{\pi R^2} e^{-\frac{b^2}{R^2}}$, where R is the spatial gluon distribution inside the proton [20], simplifies the calculations. We obtain that the F_2 structure function can be written as [17]

$$F_2(x, Q^2) = \frac{R^2}{2\pi^2} \sum_{u,d,s} \epsilon_i^2 \int_{\frac{1}{Q^2}}^{\frac{1}{Q_0^2}} \frac{d^2 r_t}{\pi r_t^4} \{C + ln(\kappa_q(x, r_t^2)) + E_1(\kappa_q(x, r_t^2))\} \ , \qquad (8)$$

5

where $\kappa_q = 4/9\kappa_G = (2\alpha_s/3R^2)\,\pi\,r_t^2\,xG_N(x, \frac{1}{r_t^2})$. This equation allows to estimate the high density corrections to the structure function in the DLA limit. Expanding the equation (8) for small κ_q, the first term (Born term) will correspond to the usual DGLAP equation in the small x region [Eq. (6)].

Before we discuss the dynamics for the gluon distribution at high densities, it is important to consider a general property of the structure function (i.e. the cross section): the black disc limit. As shown before, when the coherence length l_c considerably exceeds the diameter of a target which is at rest, the virtual photon transforms into hadron components well before the target, which implies that the small x physics probes the interaction of various hadron wave packets with a target. The geometrical limit for the cross section of virtual photon scattering off a nucleon target follows from the assumption that a target is black for the dominant hadron components in the wave function of the virtual photon. An identical assumption is often used to deduct the Froissart limit of hadron-hadron interactions at high energies.

Under the assumption that the interaction is black, we can estimate the total cross section (2) using the following approximations: $\rho(s, M^2) \propto M^2$ and $\sigma_{q\bar{q}+p}(s, M^2) = \pi R^2$. Consequently, we can get $\sigma(s, M^2) \propto R^2\,ln[(M_{max}^2 + Q^2)/(M_{min}^2 + Q^2)]$. Using that $M_{min}^2 \approx 4m_{pi}^2$ whereas $M_{max}^2 \propto s$, results that

$$F_2(x, Q^2) \propto Q^2 R^2\,ln\frac{1}{x} \; , \tag{9}$$

which is the black disc limit for the F_2 structure function. This bound, derived from a geometrical analysis, represents the maximum value allowed for the cross sections at high energies (high densities). In the next sections we will analyze the predictions of the high density approaches for the asymptotic behavior of F_2 and verify if this bound is saturated or a softer behavior is predicted.

3   The AGL High Density Approach

About seventeen years ago, Gribov, Levin, and Ryskin (GLR) [5] performed a detailed study of the high density limit of QCD in the double logarithmic approximation (DLA). They argued that the physical processes of interaction and recombination of partons become important in the parton cascade at a

large value of the parton density, and that these high density corrections could be expressed in a new evolution equation - the GLR equation. Some years ago an eikonal approach to the high density corrections was proposed in the literature by Ayala, Gay Ducati, and Levin (AGL) [6, 17] which improves the GLR approach. This approach address the high density regime from the region where perturbative QCD is valid by summing corrections to the linear DGLAP evolution equation.

As we are only interested in the solutions of the AGL equation, we refer the original papers [6, 17] for details in its derivation. Here we only present the AGL equation, which is given by

$$\frac{\partial^2 xG(x, Q^2)}{\partial y \partial \epsilon} = \frac{2\, Q^2 R^2}{\pi^2} \{C + ln[\kappa_G(x, Q^2)] + E_1[\kappa_G(x, Q^2)]\} \; , \qquad (10)$$

where C is the Euler constant, E_1 is the exponential function and the function κ_G is defined by

$$\kappa_G(x, Q^2) \equiv \frac{\alpha_s N_c \pi}{2Q^2 R^2} xG(x, Q^2) \; , \qquad (11)$$

and represents the probability of gluon-gluon interaction inside the parton cascade. Using the above definition for κ_G we can rewrite the expression (10) in a more convenient form (for fixed α_s)

$$\begin{aligned}
\frac{\partial^2 k_G(y, \epsilon)}{\partial y \partial \epsilon} + \frac{\partial k_G(y, \epsilon)}{\partial y} &= \frac{\alpha_s N_c}{\pi} \{C + ln[\kappa_G(x, Q^2)] + E_1[\kappa_G(x, Q^2)]\} \\
&\equiv F(\kappa_G) \; . \qquad (12)
\end{aligned}$$

Analyzing the structure of the Eq. (12) we see that it has a solution which depends only on y. In [6] it was shown that this solution is the asymptotic solution of the AGL equation. In this case we have that at large values of densities the asymptotic solution is given by

$$k_G^{asymp}(y) = \frac{\alpha_s N_c}{\pi} \, y \; . \qquad (13)$$

Substituting the definition of κ_G [Eq. (11)] in the above solution, in the asymptotic regime the behavior of the gluon distribution is given by (at fixed α_s)

$$xG(x, Q^2) = \frac{2N_c Q^2 R^2}{3\pi^2} \, ln\left(\frac{1}{x}\right) \; . \qquad (14)$$

7

Therefore, the gluon distribution does not saturate at small values of x, but is linearly proportional to $\ln s$ ($s \approx 1/x$). However, this behavior is softer than predicted by the DGLAP equation ($xG \propto exp[\sqrt{\ln 1/x}]$) and the BFKL equation ($xG \propto x^{-\lambda}$, $\lambda > 0$). We obtain that the gluon distribution presents a partial saturation in its behavior.

The analyzes above was made at leading order, considering a fixed coupling constant α_s. One question is the possible modifications in the asymptotic behavior associated to next-to-leading order corrections as, for instance, the running of α_s. In order to answer this question we need to extend the Refs. [6, 17] for running coupling constant, as we will do in the following.

Since the QCD coupling constant is given by $\alpha_s(Q^2) = 4\pi/(\beta_0 \epsilon)$, where $\beta_0 = 11 - 2/3 n_f$ (n_f is the number of flavors) and $\epsilon = \ln Q^2/\Lambda^2_{QCD}$, the relation between the gluon distribution and the function κ_G can be expressed by

$$xG(x, Q^2) = \frac{2Q^2 R^2}{N_c \pi} \frac{\beta_0}{4\pi} \epsilon \, \kappa_G(x, Q^2) \ . \tag{15}$$

Using this result in Eq. (10) we obtain that the AGL equation with running α_s is given by

$$
\begin{aligned}
\frac{\partial^2 k_G(y, \epsilon)}{\partial y \partial \epsilon} + \left(\frac{1}{\epsilon} + 1 \right) \frac{\partial k_G(y, \epsilon)}{\partial y} &= \frac{N_c \alpha_s(Q^2)}{\pi} \{ C + \ln[\kappa_G(x, Q^2)] \\
&+ E_1[\kappa_G(x, Q^2)] \} \\
&\equiv H(\kappa_G) \ .
\end{aligned} \tag{16}
$$

The AGL equation at fixed α_s is a direct consequence of the above equation in the limit of large ϵ. Moreover, this equation also has a solution which depends only on y. In this case we have

$$\frac{\partial k_G^{asymp}(y, \epsilon)}{\partial y} = \frac{\epsilon}{1 + \epsilon} H(\kappa_G) \tag{17}$$

with the solution

$$\int_{k_G^{asymp}(y=y_0)}^{k_G^{asymp}(y)} \frac{d\kappa'_G}{H(\kappa'_G)} = \frac{\epsilon}{1 + \epsilon} (y - y_0) \ . \tag{18}$$

8

Consequently, the asymptotic behavior of the gluon distribution is given by

$$xG(x, Q^2) = \frac{\epsilon}{1+\epsilon} \, \frac{2N_c Q^2 R^2}{3\pi^2} \, ln \left(\frac{1}{x}\right) \, . \tag{19}$$

As expected at large values of ϵ (Q^2) this solution reduces to the solution at fixed α_s. The main difference is the behavior at small values of ϵ, where the prefactor in (19) is important. However, the partial saturation of the gluon distribution is not modified by the running of α_s. It agrees with the result that the unitarity corrections are expected to be relevant before the next to leading order corrections [3].

Before we present our results and conclusions in the next section, let us discuss the generality of the above results for the gluon distribution by the comparison with the predictions of other high density approach. In contrast to the AGL formalism, the approach proposed by McLerran, Venugopalan and collaborators (MV-JKLW approach) [21, 7] address the high density regime from the nonperturbative region by developing an effective Lagrangian approach to a very dense system. In Ref. [22] the authors have considered the DLA limit of the functional evolution equation derived from this approach, obtaining an equation similar, but not identical, to the AGL equation (we refer [22] for details). One of the main results of this work was the solution of the proposed equation for large values of densities (large κ), which is given by

$$xG(x, Q^2) = \frac{N_c(N_c - 1)\pi}{2} \, R^2 \, Q^2 \, ln \left(\frac{1}{x}\right) \, . \tag{20}$$

The remarkable feature of this solution is the same Q^2 and x dependence of the AGL equation in the asymptotic regime. The difference in the prefactors is a function of the distinct normalizations and approximations used in the two approaches.

4 The Asymptotic Behavior of F_2

The main conclusion of our analysis in the previous section was the universality of the x and Q^2 dependence of the gluon distribution (the partial saturation) in the asymptotic regime of the high density QCD. In this section

we will consider the consequences of the partial saturation in the behavior of the structure function, which is experimentally measured.

Using the solution of the AGL equation in the asymptotic regime [Eq. (14)] as input in the Eq. (6) we get

$$F_2(x, Q^2) \approx \frac{\alpha_s}{\pi^3} R^2 Q^2 \, ln\left(\frac{1}{x}\right) \; . \tag{21}$$

We see that the partial saturation of the gluon distribution implies that the F_2 structure function does not saturate at small values of x, but is linearly proportional to $ln\,s$. Basically, we verify that the AGL approach, as well as the MV-JKLW approach maximizes the prediction for the F_2 structure function in the asymptotic regime of very high densities, resulting in the black disc prediction, when the leading twist relation [Eq. (6)] is used.

We now consider that the relation between the structure function and the gluon distribution is given by the Eq. (8). In the asymptotic regime (large κ_q) we obtain

$$F_2(x, Q^2) \approx \frac{R^2}{2\pi^2} \sum_{u,d,s} \epsilon_i^2 \int_{\frac{1}{Q^2}}^{\frac{1}{Q_0^2}} \frac{d^2 r_t}{\pi r_t^4} \, ln(\kappa_q(x, r_t^2)) \; . \tag{22}$$

Using the asymptotic solution of the AGL equation we can determine $\kappa_q = 4/9 \kappa_G$ at large values of densities, and so

$$F_2(x, Q^2) \approx \frac{R^2}{2\pi^2} \sum_{u,d,s} \epsilon_i^2 \int_{\frac{1}{Q^2}}^{\frac{1}{Q_0^2}} \frac{d^2 r_t}{\pi r_t^4} \, ln\left[\frac{4\alpha_s}{3} \, ln\left(\frac{1}{x}\right)\right] \; . \tag{23}$$

Therefore, considering the contribution of the higher twist terms in the relation between the structure function and the gluon distribution, we predict the following asymptotic behavior for the structure function

$$F_2(x, Q^2) \approx \frac{R^2 Q^2}{3\pi^2} \, ln\left[\frac{4\alpha_s}{3} \, ln\left(\frac{1}{x}\right)\right] \; . \tag{24}$$

We see that the inclusion of the higher twist terms implies a softer dependence of F_2 with the energy than obtained using the leading twist relation. In contrast with the leading twist prediction, in this case the black disc limit is not maximized. However, in both cases the structure function does not violate the Froissart boundary in the asymptotic regime of high density QCD. The

demonstration of the behavior (24) using the other approaches for hdQCD is an important open question. In particular, the analyzes of the predictions for F_2 obtained in Ref. [16] in the infinite momentum frame for the very high density regime would be a important check of our results. Our study demonstrates the importance of the correct connection (leading or higher twist relation) between the gluon distribution and the observables when we are analyzing the interface between perturbative and nonperturbative QCD.

From the results for the structure function in the asymptotic regime we can see that this regime is characterized by the identity

$$\frac{dF_2(x, Q^2)}{dln\, Q^2} = F_2(x, Q^2) \ , \tag{25}$$

which is an important signature of the asymptotic regime of high density QCD. This regime should be reached for the case of an interaction with nuclei at smaller parton densities than in a nucleon, since $\kappa_A = A^{\frac{1}{3}} \times \kappa_N$, where κ_N is given by the expression (11).

5 Conclusions

In this paper we have analyzed the description of the F_2 structure function in the regime of very high densities, assuming first that the leading twist relation between F_2 and xG is valid and second that this relation is modified by the higher twist terms associated to the high density corrections. In the first case we have obtained that the corresponding F_2 structure function is linearly proportional to $Q^2 R^2 \, ln\, 1/x$, which agrees with the results predicted in the black disc limit. This result is not high density approach dependent. In the second case a softer behavior is obtained. In both cases, the F_2 structure function unitarizes and, as expected by construction, the black disc boundary is not violated in the asymptotic regime of high density QCD. We verify that when the parton density is such that the proton becomes black and the interaction probability is unity, the structure function becomes proportional to Q^2 and a soft energy dependence is predicted. As a by product, we have derived the AGL equation for running α_s and obtained a signature for the asymptotic regime of QCD.

Our main conclusion is that we should be careful to relate the gluon distribution with the observables (for instance, F_2, F_2^c, heavy quark production,

...) in the high density limit, since besides the behavior of the gluon distribution, also the relation between these quantities must be modified from the usual leading twist expressions.

Acknowledgments

References

[1] Yu. L. Dokshitzer. *Sov. Phys. JETP* **46** (1977) 641; G. Altarelli and G. Parisi. *Nucl. Phys.* **B126** (1977) 298; V. N. Gribov and L.N. Lipatov. *Sov. J. Nucl. Phys* **15** (1972) 438.

[2] CTEQ Collaboration. *Rev. Mod. Phys.* **67** (1995) 157.

[3] A. H. Mueller. *Phys. Lett.* **B396** (1997) 251.

[4] E.A. Kuraev, L.N. Lipatov and V.S. Fadin. *Phys. Lett* **B60** (1975) 50; *Sov. Phys. JETP* **44** (1976) 443; *Sov. Phys. JETP* **45** (1977) 199; Ya. Balitsky and L.N. Lipatov. *Sov. J. Nucl. Phys.* **28** (1978) 822.

[5] L. V. Gribov, E. M. Levin, M. G. Ryskin. *Phys. Rep.***100** (1983) 1.

[6] A. L. Ayala, M. B. Gay Ducati and E. M. Levin. *Nucl. Phys.* **B493** (1997) 305.

[7] J. Jalilian-Marian *et al. Phys. Rev.* **D55** (1997) 5414; *Phys. Rev.* **D59** (1999) 014014; *Phys. Rev.* **D59** (1999) 014015.

[8] Y. U. Kovchegov. *Phys Rev.* **D60** (1999) 034008.

[9] M. B. Gay Ducati, V. P. Gonçalves. *Nuc. Phys.* **B557** (1999) 296.

[10] A. Kovner, J. Guilherme-Milhano, H. Weigert. *Phys Rev.* **D62** (2000) 114005.

[11] V. N. Gribov. *Sov. Phys. JETP* **29** (1969) 483.

[12] A. Rostovtsev, M. G. Ryskin, R. Engel. *Phys. Rev.* **D59** (1999) 014021.

[13] N. Nikolaev and B. G. Zakharov. *Z. Phys.* **C49** (1990) 607.

[14] A. L. Ayala, M. B. Gay Ducati and E. M. Levin. *Phys. Lett.* **B388** (1996) 188.

[15] M. Froissart. *Phys. Rev.***123** (1961) 1053.

[16] L. McLerran, R. Venugopalan. *Phys Rev.* **D59** (1999) 094002.

[17] A. L. Ayala, M. B. Gay Ducati and E. M. Levin. *Nucl. Phys.* **B511** (1998) 355.

[18] K. Golec-Biernat, M. Wusthoff. *Phys Rev.* **D59** (1999) 014017.

[19] J. Bartels, K. Golec-Biernat, K. Peters. *Eur. Phys. J.* **C17** (2000) 121.

[20] M. B. Gay Ducati, V. P. Gonçalves. *Phys. Lett.* **B487** (2000) 110.

[21] L. McLerran, R. Venugopalan. *Phys. Rev.* **D49** (1994) 2233.

[22] J. Jalilian-Marian *et al. Phys. Rev.* **D59** (1999) 034007.